Objectivity of Quantum Measurement in Many-Observer World

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The objectivity of quantum measurement is treated as an emergent phenomenon with N observers who can agree to the same result of measurement, and meanwhile, they can identify their records with each other. In this many-observer world (MOW), an objective quantum measurement is dealt with as a multipartite $[(N+1)$-body$]$ quantum correlation among the measured system and N observers when its bipartite reductions are the same classical correlations. With this conceptual clarification, we find that, an objective quantum measurement is implemented if and only if the MOW is initially factorized in a pure state and then the total system can evolve into a generalized GHZ state with respect to the orthogonal basis preferred by each observer. Especially, such objective quantum measurement is recast in ideal classical correlation when the observer world is macroscopic for $N \to \infty$.

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I. INTRODUCTION

In physics, the objectivity is used to describe the observation (or measurement) without personal bias; in principle all observers should agree to the same observation about the output from an experiment or a theoretical model. In the Copenhagen version of quantum mechanics interpretation (QMI), however, the objectivity of measurement seems not to be guaranteed since a measurement by an observer could causes an dramatical change of quantum state [1]. This is because Copenhagen interpretation treats the measuring apparatus (or observer) as a purely classical term, and thus leads to the wave-function collapse (WFC).

Many physicists [2–5], however, felt weird that the apparatus (or observer) is composed of indispensable ingredients in quantum prescription, but it does not abide by quantum mechanics. Therefore, the "built-in" interpretations are proposed without postulating the pure classicality of measuring apparatus joyed by the WFC, e.g., decoherence approaches [1, 6–9], consistent history [10, 11], and the many worlds interpretations (MWI) [12, 13]. To go much beyond these approach, quantum mechanics even was interpreted as an effective theory coming from some underlaying theory, e.g., Bohm's hidden variable approach [14, 15], 't Hooft's deterministic and dissipative theory [16–19], and Adler's trace dynamics theory [20, 21].

In this paper, we develop a "built-in" approach for quantum measurements with many (N) observers that only obeys the Schödinger equation without entailing any postulation like WFC. In the many-observer world (MOW), the measurement in quantum mechanics is treated as a dynamic process to generate the same bipartite classical correlations, which can be reduced form the $(N+1)$-body correlation of the entirety formed by the measured system plus MOW. In this sense our approach does not seems much different from the quantum Darwinism approach based on the decoherence theory, in which the each independent fractions of environment can behave as an observer [1, 6–9], but in our approach, the objectivity of quantum measurement is rigorously clarified when we further require that the consequences of measurement by an observer can be witnessed by the others, and then different observers measuring the same object can agree to the same result.

With this rigorous clarification in basic concept, we find that if and only if the initial state of the MOW is pure state and the coupling between the system and the MOW is of non-demolition, an unitary evolution of the entirety (formed by the system plus N observers) can reach a generalized GHZ state. Then, in the macroscopic limit with $N \to \infty$, its bipartite reductions become the same classical correlation with respect to the preferred basis so that an objective quantum measurement results from a dynamic process without referring the postulate of wave function collapse (WFC). This finding uncovers two facts: 1. Our approach mathematically describes the simultaneous emergence of objectivity and classicality, which is robust for the sequential measurements by different observers; it is the macroscopic characters of observers as well as the MOW that guarantee the two basis vectors of MOW correlated to different states of the system are orthogonal with each other; only when $N \to \infty$, an ideal measurement can be accomplished instantaneously; 2. Our approach are essentially "built-in" since it can not refer anything out of our quantum entirety. Here, the classical-quantum boundary distinguishing system and observers rests with the macroscopicality of MOW if it is regarded as coarse-graining of the infinitely many observers.

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II. QUANTUM MEASUREMENTS WITH TWO OR MORE OBSERVERS

In order to present our objective approach to measurement in quantum mechanics we revisit the QMI based on decoherence. Here, an quantum measurement or observation is completed by two steps: (S1) The non-demolition coupling of the system S to the apparatus (observer) D unitarily leads to a pre-measurement, a quantum entanglement between S and D with respect to a given basis $\{|s\rangle|s = 1, 2, \ldots, l\}$ for the Hilbert space of S, and the coupling prefers the specific observable $\hat{A}$ of S to be measured, where $\hat{A}|s\rangle = a_s|s\rangle$; (S2) The environment E surrounding S selects the preferred basis $|s\rangle$ so that the pre-measurement becomes a quantum measurement, which is precisely defined as a classical correlation emerging from the quantum entanglement for pre-measurement.

Let $|d\rangle$ and $|e\rangle$ be the initial states of D and E respectively. In measuring the system S initially prepared in a superposition, the state of total system (universe) $S + D + E$ will evolves into a partially entangled state

$$|\psi\rangle = (\sum c_s|s\rangle \otimes |d_s\rangle) \otimes |e\rangle, \tag{1}$$

from the initial product state $|\Psi(0) = |\psi_S(0)\rangle \otimes |d\rangle \otimes |E\rangle$. Here, $|d_s\rangle = U_s(D)|d\rangle$ is a states of D correlated to each system state $|s\rangle$ and $U_s(D)$ is the S-state dependent evolution matrix. In S2, the environment will become entangled with the system so that the total system reaches a GHZ type state

$$|\Psi\rangle = \sum c_s|s\rangle \otimes |d_s\rangle \otimes |e_s\rangle, \tag{2}$$

where the environment $|e_s\rangle = U_S(E)|e\rangle$ are orthogonal with each other, i.e., $\langle e_s|e_{s'}\rangle = \delta_{s,s'}$. By tracing over the variable of E, the correlation between S and D occurs with the representation of the reduced density matrix $\rho_{SD} = \mathrm{Tr}_E|\Psi\rangle\langle\Psi|$, that is,

$$\rho_{SD} = \sum |c_s|^2|s, d_s\rangle\langle s, d_s|, \tag{3}$$

where $|s, d_s\rangle = |s\rangle \otimes |d_s\rangle$. The above separable state implements a quantum measurement with the help of environment E. Usually, we do not need to require the states $\{|d_s\rangle|s = 1, 2, \ldots\}$ being orthogonal with each other. One could distinguish the systems states $\{|s\rangle|s = 1, 2, \ldots, l\}$ so long as $\langle d_{s'}|d_s\rangle \neq 1$, i.e, the observer's corresponding states are not identical. Of course, an ideal quantum measurement requires the observer's final states to be orthogonal, guaranteeing the objectivity of obtained results (we will prove this point in the following).

It is noticed from Eq. (2) that when $\{|d_s\rangle|s = 1, 2, \ldots\}$ as $\{|e_s\rangle|s = 1, 2, \ldots\}$ well as are orthogonal with each other, we can not mathematically distinguishes between the observers and environment since it displays a permutation symmetry for $|d_s\rangle$ and $|e_s\rangle$ exchange with each other. Thus the current decoherence approach does not clearly claim what is the boundaries among observer and the environment. With this consideration, we can replace the environment with an extra observer D'. Actually, Zurek has stressed many times that the environment in the decoherence approach has been recognized as a witness of the measurement, which essentially plays the role of another measuring device or observer; an large environment with redundancy of degrees of freedom can be divided into several portions, which could be regarded as observers.

With the above considerations we can define the quantum measurement with two or more observers where the total system is made up of a system S and two observers D and D'. Let $|d_s\rangle$ ($|d'_s\rangle$) for $s = 1, 2, ..., l(l')$ forms a basis of D (D') space of the observer 1(2). Then a two observer-quantum measurement is implement by a tripartite decomposition

$$|\Psi\rangle = \sum c_s|s\rangle \otimes |d_s\rangle \otimes |d'_s\rangle, \tag{4}$$

Due to the observer-1 measuring the system, there also exists a correlation between S and D' described with the reduced density matrix $\rho_{SD'} = \mathrm{Tr}_D|\Psi\rangle\langle\Psi|$, or

$$\rho_{SD'} = \sum |c_s|^2|s, d'_s\rangle\langle s, d'_s|. \tag{5}$$

if $D's$ states $\{|d_s\rangle|s = 1, 2, \ldots\}$ are orthogonal with each other. If the basis vectors $|s\rangle$ of S are orthogonal with each other, the two observers can compare their obtained results through a classical communication defined by the reduced density matrix

$$\rho_{DD'} = \mathrm{Tr}_S|\Psi\rangle\langle\Psi| = \sum |c_s|^2|d_s, d'_s\rangle\langle d_s, d'_s|. \tag{6}$$

We remark that: (1) in the above operation, taking trace implies that the Born rule has been used to do some average or coarse-graining; (2) the above arguments and the the following discussions can be extended to the situations with more than two observers, and the N-observer approach for quantum measurement will reveals the essential relations of the objectivity to the emergence of the classicality within the quantum world in the macroscopic limit [22]; (3) We can use the tripartite correlations (3), (5) and (6) to define an objective quantum measurement, and thus the non-orthogonal vectors can not be distinguished objectively. In fact, if $\{|s\rangle|s = 1, 2, ..l\}$ were not orthogonal with each other, then there would not be perfect classical correlation like Eq. (6), namely, the reduced density matrix

$$\tilde{\rho}_{DD'} = \rho_{DD'} + \sum_{s \neq s'} c_s c^*_{s'}|d_s, d'_{s'}\rangle\langle d_s, d'_{s'}|\langle s|s'\rangle$$

contains a coherent term $\tilde{\rho}_{DD'} - \rho_{DD'}$, which will blur the perfect correlation of two observers to do the mutual consulting about their observations. Thereafter we schematically illustrate a quantum entanglement (or a imperfect classical correlation) in Fig. 1(a) where the correspondence of the states $|d_s\rangle$ to $|s\rangle$ with a circle of blurred

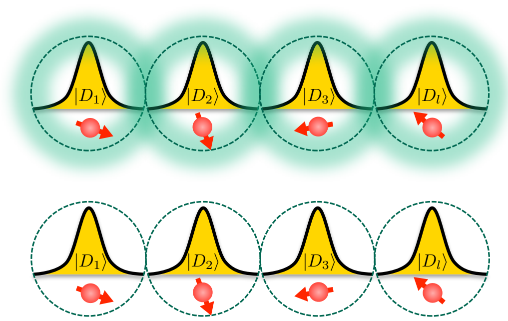

Figure 1: (a)Premeasurement of a high spin like in the Stern-Gerlach experiment represented by an quantum entanglement between the splitting spatial wave packet states $|d_s\rangle$ and the spin states $|s\rangle$ (denoted by the blurred circle). (b) The quantum measurement after decoherence. The states $|s\rangle$ and $|d_s\rangle$ are not entangled but become classically correlated, as clearly depicted by the solid line circle.

lines. After the decoherence induced by environment or another observer, the emergent classical correlation is denoted by a circle of solid line in Fig. 1(b).

III. OBJECTIVITY OF QUANTUM MEASUREMENT

Without loss of generality, we consider the objectivity in a "two-observer" measurement and the initial state is represented by a density matrices ρ (ρ may be mixed). We first define what is an objective quantum measurement for the case with two observers; the "many-observer" generalization is straightforward.

Let $\mathcal{H}_S$, $\mathcal{H}_D$ and $\mathcal{H}_{D'}$ be the Hilbert space of the system S and two observers D and D' respectively. For a given basis vectors $\{|s\rangle|s = 1, 2, \ldots, l\}$ of $\mathcal{H}_S$ to be measured, if the state ρ of the three body entirety satisfies

$$\mathrm{Tr}_{D'}\rho = \sum_{s=1}^{l} |c_s|^2 |s,\, d_s\rangle\langle s,\, d_s|, \tag{7a}$$

$$\mathrm{Tr}_{D}\rho = \sum_{s=1}^{l} |c_s|^2 |s,\, d'_s\rangle\langle s,\, d'_s|, \tag{7b}$$

$$\mathrm{Tr}_{S}\rho = \sum_{s=1}^{l} |c_s|^2 |d_s,\, d'_s\rangle\langle d_s,\, d'_s|, \tag{7c}$$

namely, the classical correlation of $S-D$, $S-D$' and $D-D'$ given by the bipartite reductions of ρ are the same with respect to the preferred basis $\{|s\rangle|s = 1, 2, \ldots, l\}$, then we say the two observers can together accomplish an objective measurement or the measurement by different observers possesses an objectivity.

Intuitively, the first two lines of the above three equations show that two observers can see the same results of observation, while the correlation between D and D'

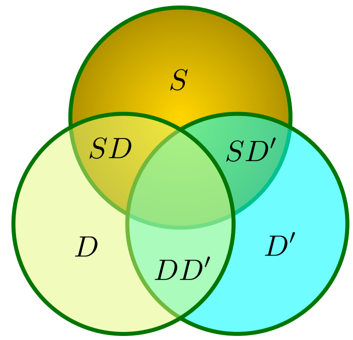

Figure 2: Correlation of three body S, D, D'. The overlapped areas represents their correlation.

in the third line acquires a mutual consultation for their observations. Schematically, we illustrate the quantum objectivity in the Fig. 2: the overlapped areas represent the quantum entanglements pair by pair among S, D and D'. In areas of $S \cap D$, $S \cap D'$ and $D \cap D'$, there exist three bipartite correlations. These diagrammatically shown results mean that in the areas $S \cap D \cap D'$, where these correlations can become perfectly classical (we will prove as follows), the two observers can see the same results and also able to compare them with each other.

In the above definition of quantum objectivity, we do not priori request that states $\{|d_s\rangle|s = 1, 2, \ldots\}$, $\{|d'_s\rangle|s = 1, 2, \ldots\}$ as well as $\{|s\rangle|s = 1, 2, \ldots\}$ are orthogonal with each other. We will prove that their orthogonality can be implied by an additional requirement: by preparing the observers initially in proper states, the state of the total system will evolve into a pure state in the form of tripartite Schmidt decomposition with respect to the orthogonal states $|d_s\rangle$, $|d'_s\rangle$ and $|s\rangle$ for $s = 1, 2, \ldots$, which gives perfect classical correlation of two parties among s, D and D' formally defined by Eqs. (7a-7c). In this situations, the objective quantum measurement is so ideal that the maximum information can be extracted.

To make the form of ρ concrete as well as the interaction of S with D and D', we will prove the following propositions.

Proposition 1: Satisfying Eqs. (7a-7c), the density operator ρ generally reads

$$\rho = \sum_{sr} p_{sr} |s,\, d_s, d'_s\rangle\langle r,\, d_r, d'_r|, \tag{8}$$

where $p_{ss}= |c_s|^2$, i.e.,

$$\rho = \sum |c_s|^2 |s,\, d_s, d'_s\rangle\langle s,\, d_s, d'_s|$$

$$+ \sum_{s\neq r} p_{sr} |s,\, d_s, d'_s\rangle\langle r,\, d_r, d'_r|,$$

For $s \neq r$, there is no any special constraints on p_{sr}, and they can take arbitrary complex numbers, as long as ρ is positive semi-definite.

The proof of *Proposition 1* is given in the appendix A, and now we only demonstrate its implications with an example :

$$\rho = |\alpha|^2|0,0,0\rangle\langle 0,0,0| + |\beta|^2|1,1,1\rangle\langle 1,1,1| \\ + \xi|0,0,0\rangle\langle 1,1,1| + \xi^*|1,1,1\rangle\langle 0,0,0| \quad (9)$$

Obviously, it satisfies Eqs. (7a-7c). When $\xi = \alpha\beta^*$, $\rho = |\Psi\rangle\langle\Psi|$ is just the GHZ state with $|\Psi\rangle = \alpha|0,0,0\rangle + \beta|1,1,1\rangle$ and it thus implement an objective quantum measurement satisfying $\mathrm{Tr}_x\rho = |\alpha|^2|0,0\rangle\langle 0,0| + |\beta|^2|1,1\rangle\langle 1,1|$ for $x = S, D, D'$. We notice that, in this example, because $|1\rangle$ and $|0\rangle$ are orthogonal with each other, the measurement is so ideal that the pointer states can be well distinguished. However, *Proposition 1* is not specific for orthogonal states of D and D', but the orthogonal basis of $|d_s\rangle$ and $|d_{s'}\rangle$ can guarantee the perfect objectivity of the measurement as we will show as follows.

Proposition 2: We assume that the tripartite state Eq. (8) comes from the following unitary evolution

$$\rho = U\rho(0)U^\dagger \equiv U[\rho_S(0)\otimes\rho_{DD'}(0)]U^\dagger, \quad (10)$$

namely, the final state ρ is the result of the dynamic evolution driven by the interaction of S to D and D', then the objectivity of quantum measurement implies:

1) The initial state of observer $\rho_{DD'}(0) := \rho_D(0)\otimes\rho_{D'}(0)$ is a pure state,

2) $\{|s\rangle\}$, $\{|d_s\rangle\}$ and $\{|d'_s\rangle\}$ are respectively orthogonal basis sets.

Proof: Since Eq.(8) should apply for any initial state, we choose $\rho_s(0) = |s_0\rangle\langle s_0|$. In this case, we have $l = 1$ and ρ in Eq.(8) should only contain one term, i.e.,

$$\rho = |s_0,\, d_{s_0},\, d'_{s_0}\rangle\langle s_0,\, d_{s_0},\, d'_{s_0}|. \quad (11)$$

Thus, ρ is a pure state and so is $\rho(0) = U^\dagger\rho U$. Since $\rho(0) = \rho_s(0)\otimes\rho_D(0)\otimes\rho_{D'}(0)$, its purity also implies that $\rho_D(0)$ and $\rho_{D'}(0)$ must be pure states.

Next we choose a pure state $\rho_s(0) = |\phi\rangle\langle\phi|$ as the initial state of S. The state of the total system ρ after premeasurement,

$$\rho = U|\phi\rangle\langle\phi|\otimes\rho_{DD'}(0)U^\dagger,$$

is also pure since we have proved that $\rho_{DD'}(0)$ is a pure state. Thus ρ can be rewritten as $\rho = |\Psi\rangle\langle\Psi|$. For a general pure state $|\Psi\rangle = \sum_{s_1,s_2,s_3}\alpha_{s_1s_2s_3}|s_1, d_{s_2}, d'_{s_3}\rangle$ (here $\{|s_1\rangle\}, \{|d_{s_2}\rangle\}, \{|d'_{s_3}\rangle\}$ do not have to be orthogonal sets), we have

$$\rho = \sum C_{s_1s_2s_3,r_1r_2r_3}|s_1, d_{s_2}, d_{s_3}\rangle\langle r_1, d_{r_2}, d_{r_3}|,$$

where $C_{s_1s_2s_3,r_1r_2r_3} = \alpha_{s_1s_2s_3}\alpha^*_{r_1r_2r_3}$. Comparing with the form of Eq. (8), we have $C_{s_1s_2s_3,r_1r_2r_3} \neq 0$ only when $s_1 = s_2 = s_3$ and $r_1 = r_2 = r_3$. To satisfy this condition, the pure state $|\Psi\rangle$ must have the form of

$$|\Psi\rangle = \sum_s c_s|s, d_s, d'_s\rangle. \quad (12)$$

Tracing over the degree of freedom of the first observer, we have

$$\mathrm{Tr}_D|\Psi\rangle\langle\Psi| = \sum_{sr} c_s c_r^*\langle d_r|d_s\rangle|s, d'_s\rangle\langle r, d'_r|. \quad (13)$$

Comparing it with Eq.(7b), we obtain

$$\langle d_r|d_s\rangle = \delta_{sr}, \quad (14)$$

namely, the basis $\{|d_s\rangle\}$ is orthogonal. With the same reason, $\{|d'_s\rangle\}$ is also orthogonal. The above argument can also be used to proved that $\{|s\rangle\}$ is orthogonal. ∎

According to *Proposition 2* proved above, to realize an objective quantum measurement we can prior request the total system is initially prepared in a pure state. Then we can explicitly determine the form of this pure state and its corresponding coupling to carry out the dynamics of the objective quantum measurement. The result can be claimed as follows.

Proposition 3: If and only if the pure state of the total system after a measurement is an tripartite Schmidt decomposition

$$|\Psi\rangle = \sum c_s|s\rangle\otimes|d_s\rangle\otimes|d'_s\rangle, \quad (15)$$

with respect to the given preferred basis $\{|s\rangle|s = 1, 2, \ldots, l\}$ of $\mathcal{H}_S$, then it can implement an objective quantum measurement satisfying Eqs. (7a-7c) with $\{|d_s\rangle|s = 1, 2, \ldots\}$ and $\{|d'_s\rangle|s = 1, 2, \ldots\}$ orthogonal with each other.

The sufficiency of the above proposition is obvious. We calculate the reduced density matrices of Eq. (15) directly, and they do have the form of Eqs. (7a-7c). The proof of the necessity is given in the appendix B . We remark that the above *Proposition 1* can be regarded as the generalization of *Proposition 3* into the case with the initial state being mixed where the tripartite Schmidt decomposition is replaced by the density matrix.

It follows from the above *Proposition 3* that the tripartite Schmidt decomposition of the final state is necessary and sufficient to realize an objective quantum measurement. To generate such GHZ type state from a product state with respect to the preferred basis $\{|s\rangle|s = 1, 2, \ldots, l\}$, defined by the system Hamiltonian $\hat{H}_S$: $\hat{H}_S|s\rangle = E_s|s\rangle$, the coupling Hamiltonian $\hat{\mathcal{H}} = \hat{H}_S + \hat{H}_{SD} + \hat{H}_{SD'}$ should be of the non-demolition type. Here, the coupling parts $\hat{H}_{SD}$ and $\hat{H}_{SD'}$ commute with each other, and $[\hat{H}_S, \hat{H}_{SD}] = 0$, $[\hat{H}_S, \hat{H}_{SD'}] = 0$,. Thus $\hat{H}_{SD}$, $\hat{H}_{SD'}$and $\hat{H}_S$ have the common eigen-vectors $\{|s\rangle|s = 1, 2, \ldots, l\}$, i.e.,

$$\hat{H}_{SD}|s\rangle = h_s(D)|s\rangle, \qquad \hat{H}_{SD'}|s\rangle = h_s(D')|s\rangle$$

which defined the basis vectors of D and D'

$$|d_s\rangle = \exp[-ih_s(D)\,t]|d\rangle, \\ |d'_s\rangle = \exp[-ih_s(D')\,t]|d'\rangle,$$

Actually, it is shown from the above proposition and its deduction that Eqs. (7a-7c) define a quantum non-demolition (QND) measurement, that preserve the physical integrity as well as the objectivity: If one measures the system with respect to the very preferred basis $|s\rangle$, the measurement does not change the probabilities of the system in $|s\rangle$, while only changes the off-diagonal terms of the reduced density matrix; the subsequent measurement will acquire the same probability distribution. Furthermore, to stress how the Eqs. (7a-7c) explicitly reveals the objectivity of the quantum measurement, we consider a special case that the observer D' cannot distinguish two system states $|s=1\rangle$ and $|s=2\rangle$. That means the observer has the same response for measuring the different states. Namely, the unitary transformation $U(D')$ for measuring cannot split the observer's state according to the superpositions of $|s'=1\rangle$ and $|s'=2\rangle$, i.e.,

$$U(D)|s'=1\rangle\otimes|d'\rangle = |1, d_1'\rangle$$
$$U(D)|s'=2\rangle\otimes|d'\rangle = |2, d_1'\rangle$$

It leads to the tripartite Schmidt decomposition

$$|\Psi\rangle = (c_1|1,\, d_1\rangle + c_2|2,\, d_2\rangle)\otimes|d_1'\rangle + \sum_{s=3}^{l} c_s|s,\, d_s,\, d_s'\rangle.$$

which gives the reduced density matrix of S with D as

$$\rho_{SD} = |c_1|^2|1,\, d_1\rangle\langle 1,\, d_1| + |c_2|^2|2,\, d_2\rangle\langle 2,\, d_2| + c_1c_2^*|1,\, d_1\rangle\langle 2,\, d_2| + \sum_{s=3}^{l}|c_s|^2|s,\, d_s\rangle\langle s,\, d_s|$$

This implies that without the witness (distinguishing states 1 and 2) of D', it is impossible to realize a perfect classical correlation between S and D.

Our proposed objectivity of quantum measurement is actually equivalent to the unobservability of the world splitting in MWI. In DeWitt's model [13] of MWI , the measurement with two observers can be described by a final state

$$|\Psi\rangle = \sum_s c_s|s\rangle|A_1 - s\rangle \otimes |A_2 - s,\, B - gA_1\rangle$$

where $|s\rangle$ can refer to a high spin state in the generalized Stern-Gerlach experiment, which moves the pointer state of the observer-1 from $|A_1\rangle$ to $|A_1 - s\rangle$. The second observer has two parts of memory, and first part A_2 records the high spin by moving from $|A_2\rangle$ to $|A_2 - s\rangle$ while the second part B measures the first observer by moving from $|B\rangle$ to $|B - gA_1\rangle$. We can imagine $|A_1\rangle$ and $|B\rangle$ are the spatial wave packets center in A and B respectively (see Fig. 3). According to DeWitt , who used the "many-worlds" to vividly rename Hugh Everett's "relative state formulation", when wave packets $|A_1 - s\rangle$ ($|A_2 - s\rangle$) are so narrow that $\langle A_j - s'|A_j - s\rangle \approx \delta_{ss'}$ for $j = 1, 2$, the pointer state $|A_j - s\rangle$ can distinguish different spins so that the observer $(j = D, D')$ in the s-branch $|s\rangle|A_1 - s\rangle \otimes |A_2 - s,\, B - gA\rangle$ can not see $|s'\rangle|A_1 - s'\rangle \otimes |A_2 - s',\, B - gA_1\rangle$ for $s \neq s'$.

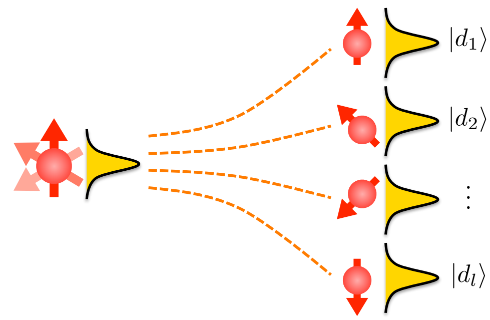

Figure 3: The measurement of the particle spin. The spatial wave packet is utilized as the apparatus. The spin degree of freedom is entangled with the spatial wave packet, and the "world" is splitted into many branches.

In our approach of objective quantum measurement, when the width of the wave packets is very small, the vectors represented by narrow wave packets will be orthogonal with each other. In this sense, we can obtain the classical correlations, such as

$$\rho(s, A_1, A_2) = \sum_s |c_s|^2|s, A_1 - s, A_2 - s\rangle\langle s, A_1 - s, A_2 - s|$$
$$\rho(s, A_i) = \sum_s |c_s|^2|s, A_i - s\rangle\langle s, A_i - s|, \qquad (i = 1, 2)$$

These classical correlation clearly defines an objective quantum measurement of high spin. It is worthy to point out that the above arguments of objective measurement could not refer to the MWI, but we can gain the same conclusion as that in MWI.

IV. INFORMATION TRANSFER, LOCALITY RECOVERY AND ORTHOGONALITY

We can examine the significance of the above propositions from the point of view from information theory.

We first notice that *Proposition 1* is not specific for orthogonal states, and the non-orthogonal $|d_s\rangle$ and $|d_{s'}\rangle$ can not perfectly guarantee the objectivity of the measurement. Actually, the quantum measurement can be understood as a procedure to extract information from the measured system by observer, and an ideal measurement can maximize the the extracted information. Here, for a reduced state

$$\rho_s = \mathrm{Tr}_{DD'}(\rho) = \sum_{s=1}^{l}|c_s|^2|s\rangle\langle s|$$

of system, the information entropy of the measurement

$$H(\rho_s|S) = -\sum\langle s|\rho_s|s\rangle\ln\langle s|\rho_s|s\rangle = -\sum|c_s|^2\ln|c_s|^2$$

is defined [12, 13] with respect to a given orthogonal basis $S = \{|s\rangle | s = 1, 2, \ldots N\}$. When we treat the measurement as a special unitary evolution $\rho(t) = U(t)\rho(0)U^{\dagger}(t)$. Then, the measurement must decrease information entropy, i.e., $H(\rho_s(t)|S) \le H(\rho_s(0)|S)$. The information transferred can be qualified by the mutual information, e.g.,

$$I_{S:D} = H(\rho_s) + H(\rho_d) - H(\rho_{sd})$$

where $H(\rho) = -\mathrm{Tr}(\rho \ln \rho)$. For the density matrix to guarantee the quantum objectivity we have

$$I(SD;t) = H(\rho_d)$$

for $\rho_d = \sum |c_s|^2 |d_s\rangle\langle d_s|$. According to Hugh Everett and others, the maximum quantum information can transfer from the system to the observers when

$$I(SD;t) = H(\rho_s(0)|S)$$

Now we show that the quantum information transferred from the system to the observers is maximized only when $|d_s\rangle(|d_{s'}\rangle)$ are orthogonal; Furthermore, only when the initial state of the total system is a factorization

$$\rho(0) = \sum_{s,r} c_s c_r^* |s\rangle\langle r| \otimes \rho_D \otimes \rho_{D'}$$

then $p_{sr}= c_s c_r^*$, i.e., the final state of the total system is a pure state – a tripartite Schmidt decomposition

$$|\Psi\rangle = \sum c_s |s\rangle \otimes |d_s\rangle \otimes |d_s'\rangle, \tag{16}$$

Actually, for information transferring, we first calculate

$$I(SD;t) = -\mathrm{Tr}_D \rho_d \ln \rho_d = \sum_d \langle d|\rho_d \ln \rho_d |d\rangle$$

$$= \sum_s |c_s|^2 \sum_d \langle d|d_s\rangle\langle d_s| \ln \rho_d |d\rangle = \sum_s |c_s|^2 \langle d_s| \ln \rho_d |d_s\rangle$$

that is $\langle d_s| \ln \rho_d |d_s\rangle = |c_s|^2$.

This point can be proved with the help of the following theorem [23]: for any density matrix $\rho = \sum p_i \rho_i$, we have

$$H(\rho) \le \sum_i p_i \ln p_i + \sum_i p_i H(\rho_i),$$

where the equality holds if and only if the support of $\rho_i$are orthogonal to each other.

Since $\{|s\rangle\}$ is an orthogonal basis, we have $H(\rho_s) = H(\rho_{sd}) = \sum |c_s|^2 \ln |c_s|^2$, and thus the mutual information is $I_{S:D} = H(\rho_s) + H(\rho_d) - H(\rho_{sd}) = H(\rho_d)$. For $\rho_d = \sum |c_s|^2 |d_s\rangle\langle d_s|$, it follows from the above theorem that

$$\begin{aligned} H(\rho_d) &\le \sum_s |c_s|^2 \ln |c_s|^2 + \sum_s |c_s|^2 H(|d_s\rangle\langle d_s|) \\ &= \sum_s |c_s|^2 \ln |c_s|^2, \end{aligned}$$

where the equality holds if and only if $|d_s\rangle$ are orthogonal to each other. Therefore, the mutual information $I_{S:D}$ achieves its maximum only when $\{|d_s\rangle\}$ is an orthogonal set.

Next we needs to re-examine whether or not there still exist some curious quantum properties predicted according to wave function collapse or its based protocols in quantum information. The new starting point is our objective definition of quantum measurement. For example, can an objective quantum measurement lead to the indistinguishability of the non-orthogonal states? If we want to "distinguish" two non-orthogonal states $|s_1\rangle$ and $|s_2\rangle$. Now we refine the performance to "distinguish" $|s_1\rangle$ and $|s_2\rangle$ as an objective quantum measurement to determine c_1 and c_2. Generally, suppose that the state we need to "distinguish" is $|\varphi\rangle = \sum c_s |s\rangle$, where $\{|s\rangle\}$ forms a basis (need not be orthogonal) of S, and the initial state of the two observers together is $|T\rangle = |d, d'\rangle$. Thus, the initial state of the total system reads

$$|\Psi(0)\rangle = \sum_s c_s |s\rangle \otimes |T\rangle.$$

The total system is isolated, and its time evolution obeys quantum mechanics, thus the measurement process can be described by an unitary evolution:

$$|\Psi\rangle = U|\Psi(0)\rangle = \sum_s c_s |\phi_s\rangle, \tag{17}$$

where $|\phi_s\rangle = U|s\rangle \otimes |T\rangle$. We hope $|\Psi\rangle$ can satisfy the three requirements of objectivity by Eqs.(7a-7c). It follows from $\mathrm{Tr}_{D'}|\Psi\rangle\langle\Psi| = \sum_{s=1}^{l} |c_s|^2 |s,\, d_s\rangle\langle s,\, d_s|$, that

$$\rho_s = \mathrm{Tr}_{DD'}|\Psi\rangle\langle\Psi| = \sum_{s=1}^{l} |c_s|^2 |s\rangle\langle s| \tag{18}$$

According to the Born rule, the reduced state of S after the measurement reads as $\rho_s = \mathrm{tr}_D |\Psi'\rangle\langle\Psi'|$, i.e.,

$$\rho_s = \sum_{ss'} c_s c_{s'}^* \mathrm{Tr}_D |\phi_s\rangle\langle\phi_{s'}|. \tag{19}$$

If only the coefficients of $c_s c_{s'}^*$ with $s \ne s'$ in Eq.(19) vanish, that is,

$$\mathrm{Tr}_{DD'} |\phi_s\rangle\langle\phi_{s'}| = \delta_{ss'} A_s. \tag{20}$$

we can reach the form

$$\rho_s = \sum_{ss'} |c_s|^2 \mathrm{Tr}_{DD'} |\phi_s\rangle\langle\phi_s|. \tag{21}$$

However, we will point out that if $|s\rangle$ and $|s'\rangle$ are non-orthogonal to each other, the above requirement Eq.(20) could not be satisfied. In fact, if we suppose Eq.(20) holds, we have

$$\langle\phi_{s'}|\phi_s\rangle = \mathrm{Tr}_S \mathrm{Tr}_{DD'} |\phi_s\rangle\langle\phi_{s'}| = \mathrm{Tr}_S(\delta_{ss'} A_s) = 0, \tag{22}$$

since $\delta_{ss'} = 0$ for $s \neq s'$. On the other hand, as a unitary evolution conserves the inner product, we have

$$\langle \phi_{s'} | \phi_s \rangle = \langle s' | \otimes \langle D | \cdot | s \rangle \otimes | D \rangle = \langle s' | s \rangle \neq 0, \qquad (23)$$

which leads to a conflict and thus proves our claim. Furthermore, Eq.(23) tells us that the inner product, $\langle \phi_{s'} | \phi_s \rangle$, does not depend on the measuring apparatus, or observers, or environments. Thus, we also exclude the possibility that $\langle \phi_{s'} | \phi_s \rangle$ may tend to zero under the thermodynamic limit.

Finally,we point out that the above definition of quantum measurement refined with objectivity is actually the celebration of the significance implied by DeWitt's model as well as the argument about quantum non-locality given in Ref. [24]. In Ref. [24], Frank Tipler associated measurement of two local spins in a Bell state $|B\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ with two remote local unitary transformations U_x and U_y at two positions x and y that are space-like separated, so that $[U_x, U_y] = 0$. Let $|D(x)\rangle$ and $|D(y)\rangle$ be the local state of the observers at x and y. Then we first perform the local measurement U_x and then carry out U_y. This performance gives

$$\begin{aligned} U_x U_y |B\rangle \otimes |D(x),\, D(y)\rangle = \frac{1}{\sqrt{2}} \big(&|\uparrow\downarrow\rangle \otimes |D_\uparrow(x),\, D_\downarrow(y)\rangle \\ &- |\uparrow\downarrow\rangle \otimes |D_\uparrow(x),\, D_\downarrow(y)\rangle \big) \end{aligned}$$

The performance of measurement about spins in reversed order can also result the same result, that is, $U_x U_y |B, D(x), D(y)\rangle = U_y U_x |B, D(x), D(y)\rangle$. This means that the effects of two measurements are not correlated causally and there does not exist causality of two measurements. According to Tipler, there needs the third observer who can carry out the third measurement $U(x, y)$ to compare the two local observations, and tell them whether or not they measured the spin in the same direction. Here, as we stressed, it is just this measurement that induces the classical correlations of the local spin states and local observers without any long range correlation-quantum non-locality. Let

$$\begin{aligned} U(x,y)|D_{\uparrow/\downarrow}(x)\rangle \otimes |D\rangle = |D_{\uparrow/\downarrow}(x)\rangle \otimes |D(\uparrow/\downarrow)\rangle \\ U(x,y)|D_{\uparrow/\downarrow}(y)\rangle \otimes |D\rangle = |D_{\uparrow/\downarrow}(y)\rangle \otimes |D(\uparrow/\downarrow)\rangle \end{aligned}$$

where $|D\rangle$ is the initial state of the third observer and $|D_{\uparrow/\downarrow}\rangle$ are its state correlated to $|D_{\uparrow/\downarrow}(\alpha)\rangle$ $(\alpha = x, y)$. Then it follows from $|\Psi\rangle = U(x,y)\, U(x)\, U(y) |B\rangle \otimes |D(x),\, D(y)\rangle \otimes |D(x,y)\rangle$ that

$$\begin{aligned} \rho = \mathrm{Tr}_3[|\Psi\rangle\langle\Psi|] = \frac{1}{2} \big[&|\uparrow D_\uparrow(x); \downarrow D_\downarrow(y)\rangle \langle \uparrow D_\uparrow(x); \downarrow D_\downarrow(y)| \\ &+ |\downarrow D_\downarrow(x); \uparrow D_\uparrow(y)\rangle \langle \downarrow D_\downarrow(x); \uparrow D_\uparrow(y)| \big] \end{aligned}$$

where we have defined $|\uparrow\ D_\uparrow; \downarrow\ D_\downarrow\rangle = |\uparrow\downarrow\rangle \otimes |D_\uparrow(x),\, D_\downarrow(y)\rangle$ *et al*, and $\mathrm{Tr}_3[\dots]$ means taking trace over the third observer. Obviously, due to the third observer witnessing the two local measurements at x and y and then results in a classical correlation, where the occurrence of the local classical correlation $|D_\uparrow(x)\rangle$ and $|\uparrow_x\rangle$ is obviously independent of the measurement made by the observer at y, and vice versa.

V. CLASSICALITY FROM MACROSCOPICITY: CENTRAL SPIN MODEL

In the above arguments we use the two observer measurement as an illustration. We have shown that only if $|d_s\rangle$ and $|d'_s\rangle$ for different s are orthogonal with each other, the quantum measurement is objective. Now we will prove that if they are not orthogonal with each other, we can still implements the objective quantum measurement by using N non-ideal observers in the macroscopic limit that $N \to \infty$. This means that the classicality of quantum measurement emerges from the macroscopicity of the observer if we coarse-grain the collection of these N observers as two macroscopic observers. We have perceived that Copenhagen QMI was challenged by asking where is the classical-quantum boundary, e.g., the Schrödinger's cat paradox. The following argument will answer this question in a natural way and explain what is the substantial difference between a quantum system to be measured and its observer, both of which still abode to the basic of quantum mechanics.

We need not to require a strict objectivity that N observers $D^{(1)}, D^{(2)}, \dots, D^{(N)}$ obtain the same result when they simultaneously measure the system to form entanglement

$$|\Psi\rangle = \sum_s c_s |s\rangle \otimes \prod_{j=1}^{N} |d_s^{(j)}\rangle, \qquad (24)$$

where the single particle states $|d_s^{(j)}\rangle$ $(s = 1, 2, \dots, l_j)$ are orthogonal with each other. Now we only consider the generic case that $|\langle d_s^{(j)} | d_{s'}^{(j)} \rangle| < 1$. For very large N, we can make a coarse-graining for the set of observers $D = \{D^{(1)}, D^{(2)}, \dots, D^{(N)}\}$ into two macroscopic sets, $D = \{D^{(1)}, D^{(2)}, \dots, D^{(n)}\}$ and $D' = \{D^{(n+1)}, D^{(2)}, \dots, D^{(N)}\}$. If n is also macroscopically large, the $N+1$-particle state (24) can resort to the Schmidt decomposition (15) by writing down

$$|d_s\rangle = \prod_{j=1}^{n} \otimes |d_s^{(j)}\rangle, \qquad |d_s\rangle = \prod_{j=n+1}^{N} \otimes |d_s^{'(j)}\rangle \qquad (25)$$

Then it is easy to show that, in most cases

$$\begin{aligned} \langle d_s | d_k \rangle &= \prod_{j=1}^{n} \langle d_s^{(j)} | d_k^{(j)} \rangle \to 0 \\ \langle d'_s | d'_k \rangle &= \prod_{j=n+1}^{N} \langle d_s^{(j)} | d_k^{(j)} \rangle \to 0 \qquad (26) \end{aligned}$$

in the macroscopic limit both $n \to \infty$ and $N \to \infty$. Therefore, with coarse-graining, we reduce the quantum measurement with N observers, into the two-observer measurement. It is worthy to emphasize the objectivity of N-observer measurement is guaranteed by the macroscopicity of the two "effective" observers D and D' from coarse-graining, which results in Eq. (26), and in turn

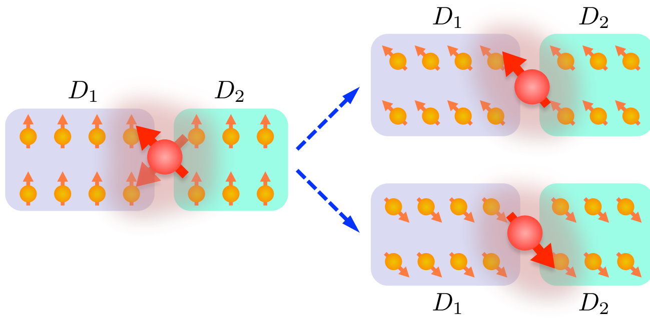

Figure 4: Central spin model. The spin environment is divided into two parts as two macroscopic devices D_1 and D_2. In the splitted two worlds after measurement, even if the states of each single spin are not orthogonal $\langle\uparrow|[R_i^{(\mathrm{g})}(t)]^\dagger\cdot R_i^{(\mathrm{e})}(t)|\uparrow\rangle\neq 0$, the two states of $D_{1(2)}$ are still nearly orthogonal, i.e., $\langle D^{(\mathrm{g})}(t)|D^{(\mathrm{e})}(t)\rangle\simeq 0$ [see Eq. (32)], which guarantees the objectivity of the macroscopic measurements by D_1 and D_2.

gives a perfect tripartite classical correlation. In this sense, we can safely say the objectivity emerges in a quantum word because its observers possess the macroscopicity in large N limit, namely, the observers consists of macroscopically many blocks, each of which behaves as an observer, and need not to entangle with the system perfectly. In short, the difference between the system and observer lies on the macroscopicity of the observers.

In the following, we use a central spin model to demonstrate the objectivity in quantum measurement [7, 22, 25]. We show that the condition of objectivity is more feasible to be satisfied when the measuring devices consist macroscopically many degrees of freedom.

The quantum system to be measured is a central spin which has two states $|\mathrm{e}\rangle$ and $|\mathrm{g}\rangle$. The central spin is surrounded by another N spin-$\frac{1}{2}$ particles, and the Hamiltonian of the total system reads

$$\hat{\mathcal{H}}=E|\mathrm{e}\rangle\langle\mathrm{e}|+\sum_{i=1}^{N}(\omega_i\hat{\sigma}_i^z+g_i\hat{\sigma}_i^x)+|\mathrm{e}\rangle\langle\mathrm{e}|\sum_{i=1}^{N}\eta_i\hat{\sigma}_i^z, \quad (27)$$

where $\hat{\sigma}_i^z=|\uparrow\rangle_i\langle\uparrow|-|\downarrow\rangle_i\langle\downarrow|$ and $\hat{\sigma}_i^z=|\uparrow\rangle_i\langle\downarrow|+|\downarrow\rangle_i\langle\uparrow|$ are the Pauli matrices for the i-th spin.

The N spins in the "environment" are separated into two groups denoted as D_1 and D_2 (see Fig. 4), which contains N_1 and N_2 spins respectively (In the above discussions, we have$N_1+N_2=N$). We have their Hamiltonians as,

$$\hat{H}_{D1}=\sum_{i=1}^{N_1}(\omega_{1,i}\hat{\sigma}_{1,i}^z+g_{1,i}\hat{\sigma}_{1,i}^x),$$
$$\hat{H}_{D2}=\sum_{j=1}^{N_2}(\omega_{2,j}\hat{\sigma}_{2,j}^z+g_{2,j}\hat{\sigma}_{2,j}^x).$$

We regard D_1 and D_2 as two macroscopic apparatus, each of which contains many even infinite degrees of freedom. An objective measurement requires that each two parts of S, D_1 and D_2 must be classically correlated, as we have discussed above. We will see that this is easily guaranteed by the macroscopicity of the two apparatus D_1 and D_2.

We assume that the initial state of the total system is

$$|\Psi(0)\rangle=\left(c_{\mathrm{g}}|\mathrm{g}\rangle+c_{\mathrm{e}}|\mathrm{e}\rangle\right)\otimes|D_1\rangle\otimes|D_2\rangle, \quad (28)$$

where $|D_{1,2}\rangle$ are the initial state of the apparatus, and

$$|D_1\rangle=\bigotimes_{i=1}^{N_1}|\uparrow\rangle,\quad |D_2\rangle=\bigotimes_{j=1}^{N_2}|\uparrow\rangle.$$

The total system evolves according to $U(t)=\exp[-i\hat{\mathcal{H}}t]$, and reaches

$$\begin{aligned}|\Psi(t)\rangle=&c_{\mathrm{g}}|\mathrm{g}\rangle\otimes|D_1^{(\mathrm{g})}(t)\rangle\otimes|D_2^{(\mathrm{g})}(t)\rangle\\&+c_{\mathrm{e}}e^{-iEt}|\mathrm{e}\rangle\otimes|D_1^{(\mathrm{e})}(t)\rangle\otimes|D_2^{(\mathrm{e})}(t)\rangle,\end{aligned} \quad (29)$$

where $|D_{1,2}^{(\mathrm{e,g})}(t)\rangle$ are the corresponding states of the two macroscopic apparatus, namely,

$$\begin{aligned}|D_1^{(\alpha)}(t)\rangle&=\bigotimes_{i=1}^{N_1}R_{1,i}^{(\alpha)}(t)|\uparrow\rangle,\\|D_2^{(\alpha)}(t)\rangle&=\bigotimes_{j=1}^{N_2}R_{2,j}^{(\alpha)}(t)|\uparrow\rangle,\end{aligned} \quad (30)$$

for $\alpha=\mathrm{g},\mathrm{e}$, and $R_{n,i}^{(\alpha)}(t)=\exp[-iH_{n,i}^{(\alpha)}t]$ is a rotating operator for the i-th spin of D_n generated from the single effective Hamiltonians

$$\begin{aligned}H_{d,i}^{(\mathrm{g})}&=\omega_{n,i}\hat{\sigma}_{n,i}^z+g_{n,i}\hat{\sigma}_{n,i}^x,\\H_{n,i}^{(\mathrm{e})}&=(\omega_{n,i}+\eta_{n,i})\hat{\sigma}_{n,i}^z+g_{n,i}\hat{\sigma}_{n,i}^x.\end{aligned} \quad (31)$$

By the requirement for the objectivity of measurement, the tripartite state Eq. (29) must have a GHZ form Eq. (15) to guarantee that each pair of S, D_1 and D_2 are classically correlated [see Eqs. (7a-7c)]. This requirement is satisfied if and only if the Loschmidt echo $E_L^{(n)}:=\left|\langle D_n^{(\mathrm{g})}(t)|D_n^{(\mathrm{e})}(t)\rangle\right|=0$ for $n=1,2$ [25]. In the above example, it has a factorized form,

$$\begin{aligned}E_L^{(n)}:=&\left|\langle D_n^{(\mathrm{g})}(t)|D_n^{(\mathrm{e})}(t)\rangle\right|=\prod_{i=1}^{N_n}\left|\langle\uparrow|[R_{n,i}^{(\mathrm{g})}(t)]^\dagger\cdot R_{n,i}^{(\mathrm{e})}(t)|\uparrow\rangle\right|\\=&\prod_{i=1}^{N_n}(1-\sin^2\mu_{n,i}^{(\mathrm{e})}t\cdot\sin^2\phi_{n,i}^{(\mathrm{e})})(1-\sin^2\mu_{n,i}^{(\mathrm{g})}t\cdot\sin^2\phi_{n,i}^{(\mathrm{g})}),\end{aligned} \quad (32)$$

where we denote

$$\begin{aligned}\mu_{n,i}^{(\mathrm{e})}&=[(\omega_{n,i}+\eta_{n,i})^2+g_{n,i}^2]^{\frac{1}{2}}, & \sin\phi_{n,i}^{(\mathrm{e})}&=\frac{g_{n,i}}{\mu_{n,i}^{(\mathrm{e})}},\\\mu_{n,i}^{(\mathrm{g})}&=[\omega_{n,i}^2+g_{n,i}^2]^{\frac{1}{2}}, & \sin\phi_{n,i}^{(\mathrm{g})}&=\frac{g_{n,i}}{\mu_{n,i}^{(\mathrm{g})}}.\end{aligned} \quad (33)$$

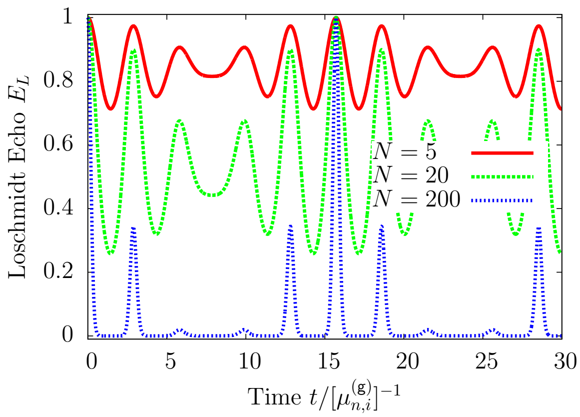

Figure 5: The decay of the Loschmidt echo E_L with time t [see Eq. (32)]. We set $\mu_{n,i}^{(\mathrm{g})} = 1$ as the energy unit. Other parameters are $\mu_{n,i}^{(\mathrm{e})} = 1.2$, $g_{n,i} = 0.2$. When the particle number N becomes large, The Loschmidt echo E_L quickly decays to zero, and only revivals at certain time, which becomes negligible when $N \to \infty$.

In the above product Eq. (32), each term is no greater that 1. Therefore, in the thermodynamic limit $N_{1,2} \to \infty$, i.e., both D_1 and D_2 consist infinite spins the Loschmidt echo automatically decrease to zero within a quite short time (see Fig. 5). When $F \simeq 0$, each two of the tripartite system S, D_1 and D_2, become classically correlated as we mentioned before [see Eqs. (7a-7c)]. At this moment, we can say that the measurement results obtained by D_1 and D_2 are objective, because they can check their results with each other and reach an agreement.

It should be noticed that in the above example, even if the states of each single spin are not orthogonal $\langle\uparrow|[R_i^{(\mathrm{g})}(t)]^\dagger \cdot R_i^{(\mathrm{e})}(t)|\uparrow\rangle \neq 0$, the two states of $D_{1(2)}$ are still nearly orthogonal, i.e., $\langle D_n^{(\mathrm{g})}(t)|D_n^{(\mathrm{e})}(t)\rangle \simeq 0$ [see Eq. (32)], and this orthogonality naturally becomes exact in the thermodynamic limit $N_{1,2} \to \infty$. That means, the objectivity of the measurement is guaranteed by the macroscopicity of the measuring devices D_1 and D_2.

VI. CONCLUSION AND REMARKS

In this paper, we shows that, when two (or more) observers classically correlated to the same preferred basis of the system, the quantum measurement is thought to be objective. For a two(or many)-observer quantum measurement, two observers are also required to compare their observations with each other. This comparison is implemented by some communication, which is also exactly described as the inter-observer classical correlations. By refining quantum measurement by stressing objectivity, the quantum puzzles, such as the EPR paradox, *et al*, no longer emerge as the very nature of quantum mechanics. We have presented three mathematical propositions to support our conclusion. Here, the quantum locality is restored by modeling two/many observers according to quantum mechanics without any classical ingredient needed.

The emergence of classicality in quantum measurement is closely related to the defined objectivity we require for refining quantum measurement. The classical reality also is clearly described by the triple correlation in avoiding the abstract concepts of information theory. It is selected by the environment from the quantum world to survive as an objective existence. In our approach we only single out the measured system from the quantum world, and all others including observers and environments are placed coequally. Undoubtably, these findings shows that some of quantum puzzles only due to the vague definition of measurement in quantum mechanics, which was obviously blurred by some purely imaginary issues, such as the wave function collapse. In this sense our approach is just accommodated by MWI where each world branch exists democratically, and thus each object in this branch also exists equally.

These arguments in favor of the hypothesis that the observers and environments are macroscopic. We have illustrated this observation with the central spin model. Here, we can also envision the N-observer world is grouped as one (or several) macroscopic observer(s), each of which also contains macroscopically many individuals and possess effectively orthogonal basis. This idea well accommodates to the quantum Darwinism interpretation (QDI) of quantum mechanics by Zurek et al [9, 26]. Here, the environment with redundant memory can behaves an witness to acquire the pointer states without disturbing the classical reality of these states. One can regard our macroscopic observer as the redundant environment in QDI, and only certain stable states can be measured repeatedly. Those states with more quantum coherence are obviously unstable in this environment. In this means, the QDI actually gives a preliminary supporting evidence for our approach.

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Appendix A: Requirement of objectivity

Proposition: If the two-body reduced density operators of the universe (composed by S, D and D') satisfies

$$\mathrm{Tr}_{D'}\rho=\sum_{s=1}^{l}|c_s|^2|s,\,d_s\rangle\langle s,\,d_s|,\tag{A1}$$

$$\mathrm{Tr}_{D}\rho=\sum_{s=1}^{l}|c_s|^2|s,\,d'_s\rangle\langle s,\,d'_s|,\tag{A2}$$

$$\mathrm{Tr}_{S}\rho=\sum_{s=1}^{l}|c_s|^2|d_s,\,d'_s\rangle\langle d_s,\,d'_s|,\tag{A3}$$

where $\{|s\rangle\}$, $\{|d_s\rangle\}$ and $\{|d'_s\rangle\}$ are linearly independent vector sets (not necessarily orthogonal) for $\mathcal{H}_S$, $\mathcal{H}_D$ and $\mathcal{H}_{D'}$ respectively. Then the density operator of the universe ρ must have the from of

$$\rho=\sum_{s,r}p_{sr}|s,\,d_s,\,d'_s\rangle\langle r,\,d_r,\,d'_r|.\tag{A4}$$

To prove this proposition, we first introduce the following lemma:

Lemma: If A is a positive $n\times n$ matrix and B is a nonnegative $n\times n$ matrix, and $\mathrm{Tr}AB=0$, then $B=0$.

Proof: For $B\geqslant 0$, we choose B diagnosable in a given orthogonal base $\{|n\rangle\}$, i.e.,

$$B|n\rangle=\lambda_n|n\rangle.\tag{A5}$$

Then,

$$\begin{aligned}0 &= \mathrm{Tr}AB=\sum_n\langle n|AB|n\rangle\\ &= \sum_n\lambda_n\langle n|A|n\rangle.\end{aligned}\tag{A6}$$

Since $B\geqslant 0$ implies $\lambda_n\geqslant 0$ and $A>0$ implies $\langle n|A|n\rangle>0$, we only have $\lambda_n=0$ so that $B=0$. ■

Now we come back to prove the proposition. Let vector sets $\{|s\rangle\}$, $\{|d_s\rangle\}$ and $\{|d'_s\rangle\}$ be the basis for $\mathcal{H}_S$, $\mathcal{H}_D$ and $\mathcal{H}_{D'}$ respectively. Thenρ can be generally written as

$$\rho=\sum C_{s_1s_2s_3,r_1r_2r_3}|s_1,\,d_{s_2},\,d'_{s_3}\rangle\langle r_1,\,d_{r_2},\,d'_{r_3}|,\tag{A7}$$

and the reduced state of the SD subsystem is

$$\mathrm{Tr}_{D'}\rho=\sum C_{s_1s_2s_3,r_1r_2r_3}\langle d'_{r_3}|d'_{s_3}\rangle|s_1,\,d_{s_2}\rangle\langle r_1,\,d_{r_2}|.\tag{A8}$$

Compare this equation with Eq.(A1), we have

$$\sum_{s_3r_3}C_{s_1s_2s_3,r_1r_2r_3}\langle d'_{r_3}|d'_{s_3}\rangle=\delta_{s_1r_1}\delta_{s_2r_2}\delta_{s_1s_2}\alpha_{s_1}\tag{A9}$$

and $\alpha_j=|c_j|^2$ for $l>j$; and $\alpha_j=0$ when $j>l$. Thus, hereafter we can restrict our analysis to the generic case $1\leq s_1,s_2,s_3\leq l$.

We define the $l\times l$ square matrices $\mathbf{C}(s_1s_2;r_1r_2)$ by

$$[\mathbf{C}(s_1s_2;r_1r_2)]_{s_3,r_3}=C_{s_1s_2s_3,r_1r_2r_3},$$

and another $l\times l$ matrix $\mathbf{A}$ is defined by $\mathbf{A}_{s_3r_3}=\langle d'_{r_3}|d'_{s_3}\rangle$. Notice that $\mathbf{A}$ is positive. For $\forall|\psi\rangle=\sum\lambda_s|d'_s\rangle$, we have $0\leq\langle\psi|\psi\rangle=\sum_{s,r}\lambda_r^*\cdot\mathbf{A}_{rs}\cdot\lambda_s$, and thus the above equations gives

$$\mathrm{Tr}[\mathbf{C}(s_1s_2;r_1r_2)\mathbf{A}]=\delta_{s_1r_1}\delta_{s_2r_2}\delta_{s_1s_2}\alpha_{s_1}\mathbf{1}.\tag{A10}$$

For $s_1=r_1$ and $s_2=r_2$, the matrix $\mathbf{C}(s_1s_2;s_1s_2)$ is non-negative since it is a principal sub-matrix of the non-negative matrix $C_{s_1s_2s_3,r_1r_2r_3}$. Thus, when $s_1\neq s_2$, we have $\mathrm{Tr}[\mathbf{C}(s_1s_2;s_1s_2)\mathbf{A}]=0$, which implies $\mathbf{C}(s_1s_2;s_1s_2)=\mathbf{0}$ according to the lemma. Therefore, the diagonal elements $C_{s_1s_2s_3,s_1s_2s_3}\neq 0$ only when $s_1=s_2$. With the same reason, by considering Eqs.(A2, A3) we can prove that $C_{s_1s_2s_3,s_1s_2s_3}\neq 0$ only when $s_1=s_2=s_3$.

For non-diagonal elements $C_{s_1s_2s_3,r_1r_2r_3}$, the necessary condition for non-vanishing $C_{s_1s_2s_3,r_1r_2r_3}$ is $s_1=s_2=s_3$ and $r_1=r_2=r_3$. Otherwise, for example, if $s_1=s_2=s_3$ is not satisfied, since it follows from the above discussion that $C_{s_1s_2s_3,s_1s_2s_3}=0$, the determinant of the following sub-matrix would be non-positive

$$\begin{pmatrix}C_{s_1s_2s_3,s_1s_2s_3} & C_{s_1s_2s_3,r_1r_2r_3}\\ C_{r_1r_2r_3,s_1s_2s_3} & C_{r_1r_2r_3,r_1r_2r_3}\end{pmatrix},\tag{A11}$$

but it should be non-negative because it is a principal sub-matrix of $[C_{s_1s_2s_3,r_1r_2r_3}]$. Therefore, if we denote $C_{sss,rrr}$ as p_{sr}, we will get the required form as in Eq.(A4). ■

We notice that, for a mixed state ρ, the orthogonality of $\{|s\rangle\}$, $\{|d_s\rangle\}$, $\{|d'_s\rangle\}$ are not guaranteed automatically. For example, for a tripartite state

$$\rho=\sum_s p_s|s,\,d_s,\,d'_s\rangle\langle s,\,d_s,\,d'_s|,\tag{A12}$$

we can verify

$$\begin{aligned}\mathrm{Tr}_{D'}\rho &= \sum_s p_s|s,\,d_s\rangle\langle s,\,d_s|\cdot\sum_n\langle\overline{d}'_n|d'_s\rangle\langle d'_s|\overline{d}'_n\rangle\\ &= \sum_s p_s|s,\,d_s\rangle\langle s,\,d_s|\cdot\sum_n\langle d'_s|\overline{d}'_n\rangle\langle\overline{d}'_n|d'_s\rangle\\ &= \sum_s p_s|s,\,d_s\rangle\langle s,\,d_s|,\\ \mathrm{Tr}_D\rho &= \sum_s p_s|s,\,d'_s\rangle\langle s,\,d'_s|,\\ \mathrm{Tr}_S\rho &= \sum_s p_s|d_s,\,d'_s\rangle\langle d_s,\,d'_s|.\end{aligned}$$

Here, we do not require that $\{|s\rangle\}$, $\{|d_s\rangle\}$, $\{|d'_s\rangle\}$ must be orthogonal.

Appendix B: Tripartite Schmidt decomposition

In this appendix we give the proof of *Proposition 3* in details. We consider its necessity: only if the total

system in measurement evolves into a special pure state that is the tripartite Schmidt decomposition with respect to the given preferred basis $\{|s\rangle | s = 1, 2, \ldots, l\}$ of $\mathcal{H}_S$, then an quantum measurement is objective defined by Eqs. (7a-7c) . To this end we consider a bipartite pure state $|\psi_{AB}\rangle$. If we have known $\rho_A = \mathrm{Tr}_B(|\psi_{AB}\rangle\langle\psi_{AB}|) = \sum_i |c_i|^2 |A_i\rangle\langle A_i|$ (here $\{|A_i\rangle\}$ does not have to be orthonormal), then $|\psi_{AB}\rangle$ must have the form

$$|\psi_{AB}\rangle = \sum_i c_i e^{i\theta_i} |A_i\rangle \otimes |B_i\rangle \tag{B1}$$

where $\{|B_i\rangle\}$ is an orthonormal set, and θ_i can be chosen arbitrarily. This is because the bipartite state $|\psi_{AB}\rangle$ always can be written as

$$|\psi_{AB}\rangle = \sum_{ij} \tilde{\alpha}_{ij} |A_i\rangle \otimes |\tilde{B}_j\rangle = \sum_i \alpha_i |A_i\rangle \otimes |B_i\rangle \tag{B2}$$

where $\{|\tilde{B}_i\rangle\}$ is an orthonormal basis, and

$$|B_i\rangle = \frac{\sum_j \tilde{\alpha}_{ij} |\tilde{B}_j\rangle}{\sum_j |\tilde{\alpha}_{ij}|^2}, \quad |\alpha_i|^2 = \sum_j |\tilde{\alpha}_{ij}|^2. \tag{B3}$$

Thus the reduced density matrix of system A is

$$\begin{aligned}\tilde{\rho}_A = &\sum_i |\alpha_i|^2 |A_i\rangle\langle A_i| \\ &+ \sum_{i\neq j} \alpha_i \alpha_j^* \langle B_j|B_i\rangle |A_i\rangle\langle A_j| + \mathbf{h.c}.\end{aligned} \tag{B4}$$

Since we have known $\rho_A = \sum_i |c_i|^2 |A_i\rangle\langle A_i|$, we must have $\alpha_i = c_i e^{i\theta_i}$ and $\langle B_j|B_i\rangle = 0$. Therefore $|\psi_{AB}\rangle$ has the form of Eq. (B1). Notice that here $\{|A_i\rangle\}$ does not have to be orthonormal, while $\{|B_i\rangle\}$ is orthogonal.

Now we prove the necessity of the above proposition, namely, if the bipartite correlations in $|\Psi_{SDD'}\rangle$ satisfy the classical correspondence Eqs. (7a-7c), then $|\Psi_{SDD'}\rangle$ must be a tripartite Schmidt form, and $\{|s\rangle\}$, $\{|d_s\rangle\}$, $\{|d'_s\rangle\}$ must be orthogonal basis.

Proof: We first treat S and D as a whole, thus $|\Psi_{SDD'}\rangle$ can be regarded as a bipartite state $S + D + D'$. From the above discussion and Eq. (7a), $|\Psi_{SDD'}\rangle$ must have the following tripartite Schmidt form

$$|\Psi_{SDD'}\rangle = \sum_s c_s |s,\, d_s\rangle \otimes |\tilde{d}'_s\rangle, \tag{B5}$$

where $\{|\tilde{d}'_s\rangle\}$ is an orthogonal basis, and we have absorbed the arbitrary phase in $|\tilde{d}'_s\rangle$.

Now we will prove $\{|s\rangle\}$, $\{|d_s\rangle\}$, $\{|d'_s\rangle\}$ must be orthogonal sets. With the same reason, Eq. (7b) guarantees that $|\Psi_{SDD'}\rangle$ also has the form

$$|\Psi_{SDD'}\rangle = \sum_s c_s |s\rangle \otimes |\tilde{d}_s\rangle \otimes |d'_s\rangle, \tag{B6}$$

where $\{|\tilde{d}'_s\rangle\}$ is an orthogonal basis.

The above Eqs. (B5, B6) should be equal, thus we must have

$$\begin{aligned}\sum_s c_s |s,\, d_s,\, \tilde{d}'_s\rangle &= \sum_s c_s |s,\, \tilde{d}_s,\, d'_s\rangle \\ &= \sum_{s,r,t} c_s \langle d_r|\tilde{d}_s\rangle \langle \tilde{d}'_t|d'_s\rangle \cdot |s,\, d_r,\, \tilde{d}'_t\rangle,\end{aligned}$$

which requires $\langle d_r|\tilde{d}_s\rangle\langle\tilde{d}'_t|d'_s\rangle = \delta_{rs}\delta_{ts}$. Since $\left|\langle d_r|\tilde{d}_s\rangle\right| \leq 1$, $\left|\langle\tilde{d}'_t|d'_s\rangle\right| \leq 1$, we must have $\langle d_r|\tilde{d}_s\rangle = \langle\tilde{d}'_t|d'_s\rangle = 1$ when we set $r = s = t$, that is, $|d_s\rangle = |\tilde{d}_s\rangle$, $|d'_s\rangle = |\tilde{d}'_s\rangle$. Therefore, $\{|d_s\rangle\}$, $\{|d'_s\rangle\}$ must be orthogonal sets. With the same reason, combining Eqs. (7a, 7c) we can also prove that $\{|s\rangle\}$ must be an orthogonal set. ■

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